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SIEVE METHODS AND THE TWIN PRIME CONJECTURE

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Abstract: For $n \geq 3$, let p_n denote the n^{th} prime number. Let $[\,]$ denote the floor or greatest integer function. For a positive integer m, let $\pi_2(m)$ denote the number of twin primes not exceeding m. The twin prime conjecture states that there are infinitely many prime numbers p such that $p+2$ is also prime. In this paper we state a conjecture to the effect that given any integer $a > 0$ there exists an integer $N_2(a)$ such that

$$\left[\frac{ap_{n+1}^2}{2(n+1)}\right] \leq \pi_2\left(p_{n+1}^2\right)$$

for all $n \geq N_2(a)$ and prove the conjecture in the case $a = 1$. This, in turn, establishes the twin prime conjecture.

Keywords and Phrases: Primes, Twin primes, Sieve methods.

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1. Introduction and Main Results

An integer $p \geq 2$ is called a prime if its only positive divisors are 1 and p. The prime numbers form a sequence:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47\ \ldots. \tag{1.1}$$

Euclid (300 B.C.) considered prime numbers and proved that there are infinitely many. Prime numbers are odd except 2 and the only consecutive prime numbers